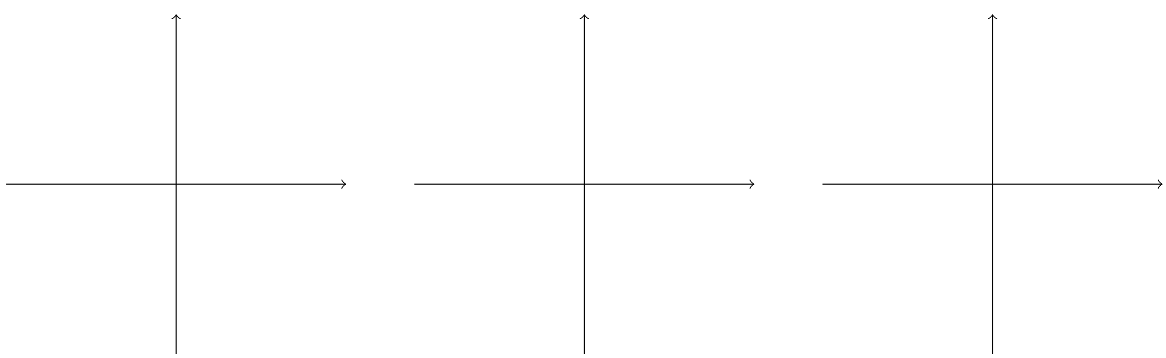
A Crash Course in Trigonometry Part II

Reference Triangles

For an angle θ in each of the second, third, or fourth quadrants, there is a corresponding angle in the first quadrant, defining a *reference triangle*. You can use the latter to calculate values of each of the trig functions for the angle θ.

1. On each of the three sets of axes below, draw a unit circle and an angle in quadrant 2, 3, and 4 respectively. Show the corresponding reference triangles in the first quadrant.



2. Using these, decide whether each of the three trig functions takes positive values and which takes negative value in each of the four quadrants. In the following table, put '+' or '-' in each of the nine spaces.

	$\sin\theta$	$\cos\theta$	$\tan\theta$
2^{nd} quad			
3^{rd} quad			
4^{th} quad			

More Special Values of the Trig Functions

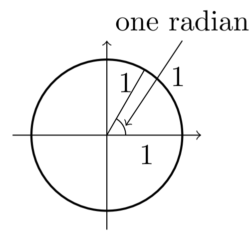
3. Using your known values of sin, cos, and tan for the angle 0°, 30°, 45°, 60°, and 90° and the idea of reference triangles above, find the values of each of the three functions for all of the following angles: 120°, 135°, 150°, 180°, 210°, 225°, 240°, 270°, 300°, 315° and 330°. Show your work. Fill in these angles and the corresponding blank spaces on the unit circle drawn a couple of pages down.

Note: The unit circle will be an extremely useful reference for you for the next few weeks. I suggest detaching it from this pack and bringing it every day with you to class.

Radians

For reasons that will become apparent next week, degrees are not a sensible measure of angles for any work with trig that involves calculus. Instead, we will measure angles in *radians*.

Definition - Radian: One *radian* is the angle in the unit circle at which the corresponding arc-length is exactly 1:



4. (a) What is the circumference of the unit circle?

   (b) If the arc-length is 1, what fraction is the arc as part of the entire circle?

   (c) If the angle is θ°, what fraction of the whole circle is that?

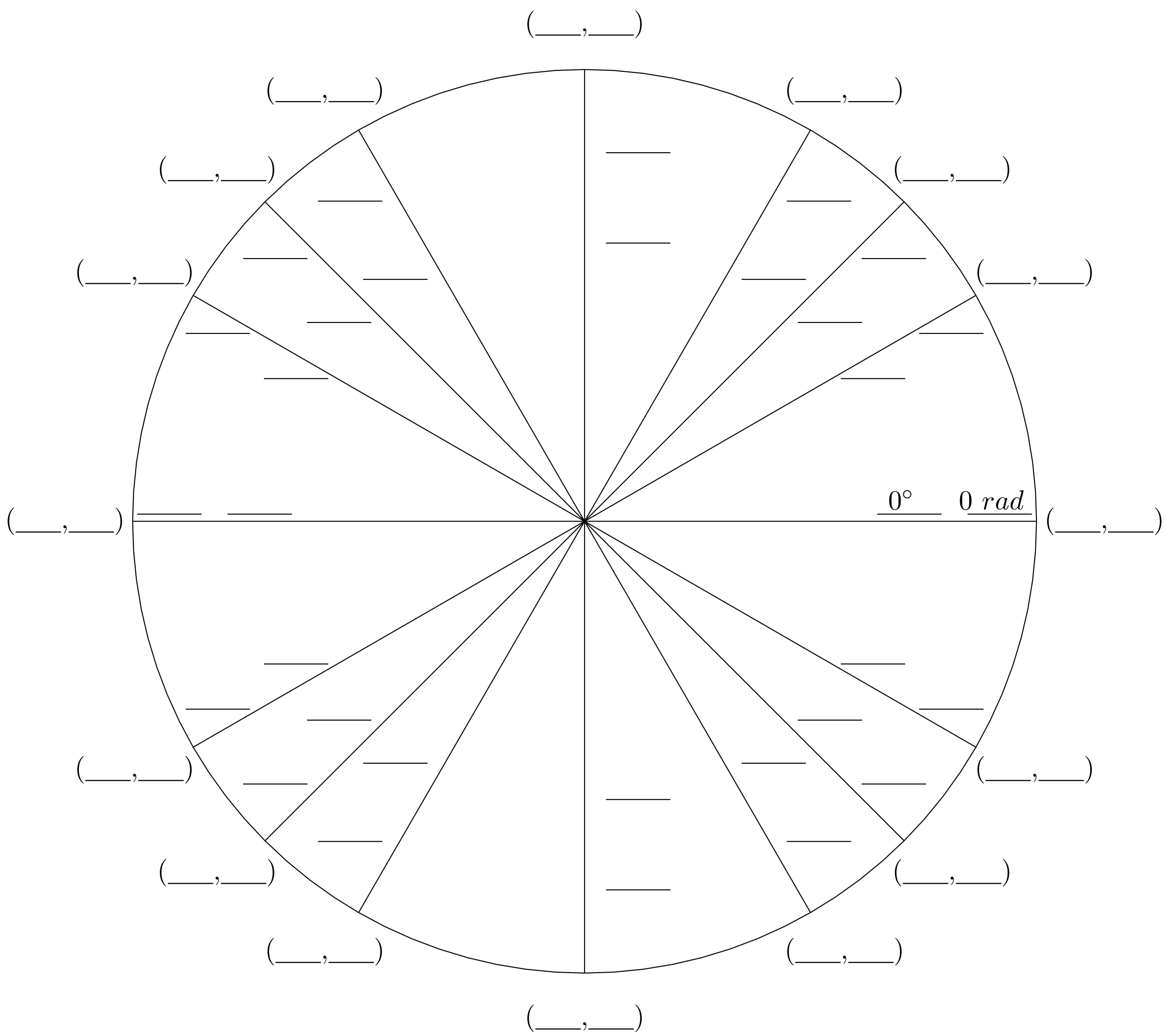
   (d) What can you say about your last two answers, and why?

   (e) Use your answer to the last question to find the size of one radian in degrees.

   (f) Lastly, fill on all the radian measures of the angles on the unit circle on the attached page.

- One radian is $\left(\qquad \right)^\circ \approx$ ______$^\circ$.
- $1^\circ = \left(\qquad \right)$ radians $\approx$ ______ radians.

The Unit Circle



Homework Problems

Solving Triangles - Trigonometry Angle Problems

Unless otherwise specified, all the below are in radians.

1. For each value of θ, draw the unit circle and the appropriate "reference triangle." Then use this to determine the *exact* values of $\sin\theta$ and $\cos\theta$: (a) $\theta = \frac{5\pi}{6}$ (b) $\theta = -\frac{\pi}{4}$ (c) $\theta = 480°$ (d) $\theta = \frac{\pi}{3} + 1,000,000\pi$

2. Suppose that the terminal side of an angle θ lies in Quadrant III and lies on the line $y = 3x$. Find $\sin\theta$. (Hint: Draw and label a picture.)

3. Let t be an angle in the first quadrant, as pictured below. Evaluate the following expressions in terms of a.

   (a) $\sin(t + 2\pi) =$ ______

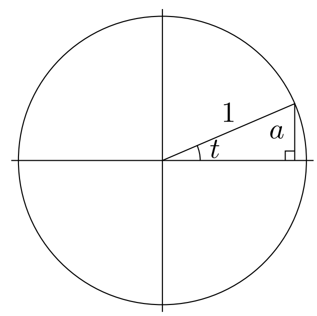
   (b) $\sin(t + \pi) =$ ______

   (c) $\cos\left(\frac{\pi}{2} - t\right) =$ ______

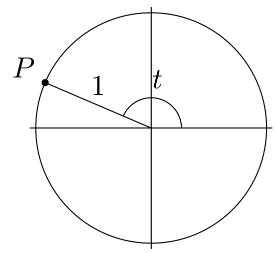
   (d) $\sin(\pi - t) =$ ______

   (e) $\sin(2\pi - t) =$ ______

   (f) $\cos\left(\frac{3\pi}{2} - t\right) =$ ______



4. True or False: In the diagram to the right, the coordinates of point P are $(-\cos t, \sin t)$.



5. Find the x and y coordinates on the unit circle determined by the following angles. If possible, find these coordinates exactly. Otherwise, approximate them.
   (a) $30°$ (b) $315°$ (c) $130°$ (d) $-240°$ (e) $1000°$

6. For what values of θ between $0°$ and $360°$ is: (a) $\sin(\theta) \geq 0$? (b) $\cos(\theta) \leq 0$? (c) $\sin(\theta) \geq \cos(\theta)$?

7. Find the exact values of the following.

$$\text{(a) } \sin\left(\frac{\pi}{3}\right) \quad \text{(b) } \cos\left(-\frac{3\pi}{4}\right) \quad \text{(c) } \sin^2(2.3) + \cos^2(2.3) \quad \text{(d) } \tan\left(\frac{7\pi}{6}\right)$$

8. The Hubble space telescope orbits the earth every 90 minutes in a near circular orbit 370 miles above the surface of the earth. (The radius of the earth is 3960 miles.)

   (a) What distance along its orbit will the Hubble telescope travel in 1 hour?

   (b) What is the radian measure of the angle through which the Hubble telescope will move after traveling 1000 miles along its orbit? (The angle is measured from the center of the earth.)