

Reminder: Inverse Functions

Defintion A function f is *invertible* if the following equivalent conditions are fulfilled:

- f is a one-to-one function;
- $x \neq y \Rightarrow f(x) \neq f(y)$;
- $f(x)$ passes the horizontal line test.

If $f^{-1}(x)$ is the inverse of f , then

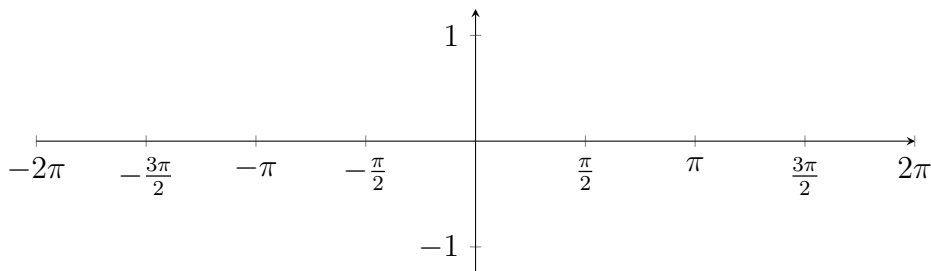
- $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1} , and $f^{-1}(f(x)) = x$ for all x in the domain of f .
- $y = f^{-1}(x) \Leftrightarrow x = f(y)$.
- $\text{Domain}(f^{-1}) = \text{Range}(f)$ and $\text{Range}(f^{-1}) = \text{Domain}(f)$.
- The graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = x$.

Inverting Trigonometric Functions

Now let's apply what we know about inverse functions in general to the specific functions $y = \sin x$ and $y = \tan x$.

sin x

- (a) On the axes below graph $y = \sin x$ on $[-2\pi, 2\pi]$.



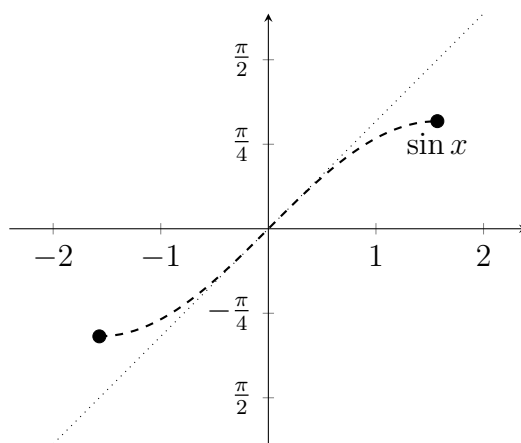
- Why is $\sin x$ not invertible on $[-2\pi, 2\pi]$?
- What is the simplest domain on which $\sin x$ is invertible?
- Let's call the inverse of $\sin x$ on that domain $\sin^{-1} x$, or $\arcsin x$. What is the domain of $\sin^{-1} x$? The range?

2. (a) What is $\sin(\sin^{-1} x)$? For which values of x is that true?

(b) What is $\sin^{-1}(\sin x)$? For which values of x is that true?

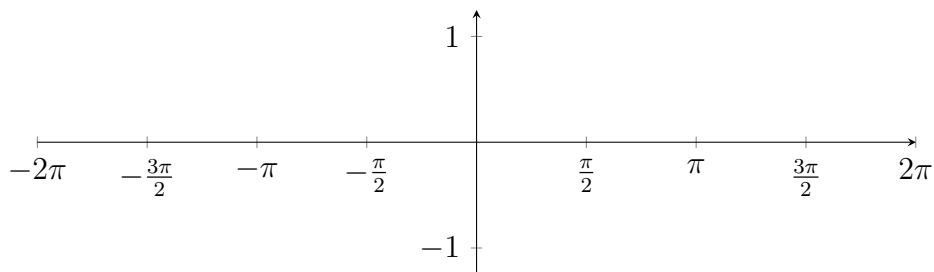
(c) If $y = \sin^{-1} x$, then $x =$ _____.

(d) The axes below show a graph of $\sin x$ on the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$. Sketch a graph of $y = \sin^{-1} x$. The line $y = x$ is shown dotted to help you.



COS x

3. (a) On the axes below graph $y = \cos x$ on $[-2\pi, 2\pi]$.



(b) Why is $\cos x$ not invertible on $[-2\pi, 2\pi]$?

(c) What is the simplest domain on which $\cos x$ is invertible?

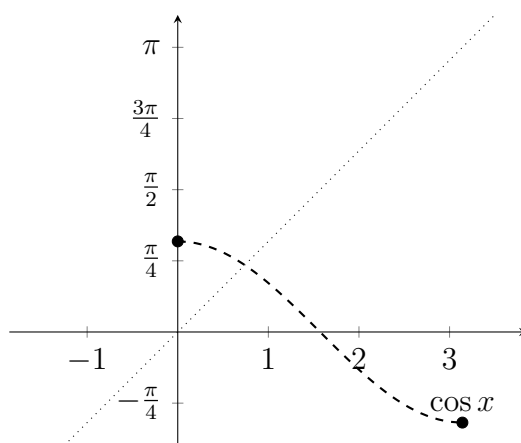
(d) Let's call the inverse of $\cos x$ on that domain $\cos^{-1} x$, or $\arccos x$. What is the domain of $\cos^{-1} x$? The range?

4. (a) What is $\cos(\cos^{-1} x)$? For which values of x is that true?

(b) What is $\cos^{-1}(\cos x)$? For which values of x is that true?

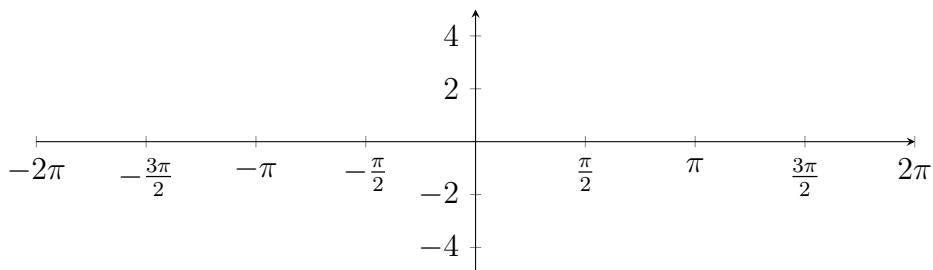
(c) If $y = \cos^{-1} x$, then $x =$ _____.

(d) The axes below show a graph of $\cos x$ on the domain $[0, \pi]$. Sketch a graph of $y = \cos^{-1} x$. The line $y = x$ is shown dotted to help you.



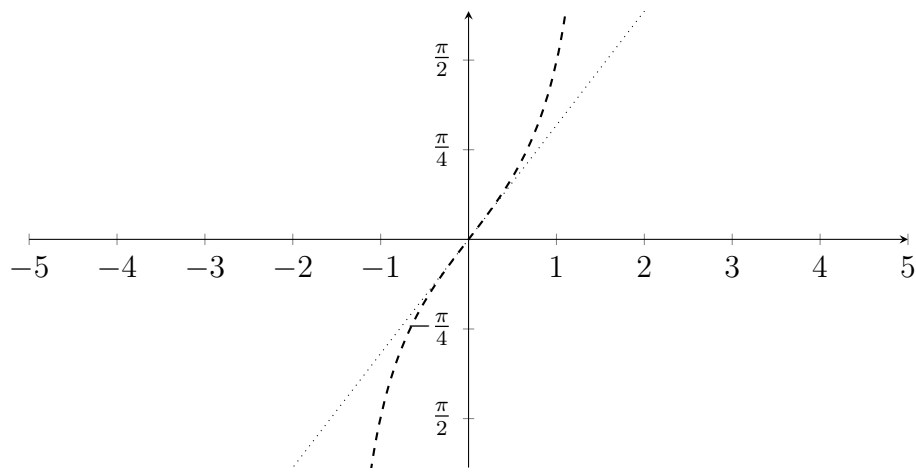
tan x

5. (a) On the axes below graph $y = \tan x$ on $[-2\pi, 2\pi]$.



(b) Why is $\tan x$ not invertible on $[-2\pi, 2\pi]$?

- (c) What is the simplest domain on which $\tan x$ is invertible?
- (d) Let's call the inverse of $\tan x$ on that domain $\tan^{-1} x$, or $\arctan x$. What is the domain of $\tan^{-1} x$? The range?
6. (a) What is $\tan(\tan^{-1} x)$? For which values of x is that true?
- (b) What is $\tan^{-1}(\tan x)$? For which values of x is that true?
- (c) If $y = \tan^{-1} x$, then $x =$ _____.
- (d) The axes below show a graph of $\tan x$ on the domain $(-\frac{\pi}{2}, \frac{\pi}{2})$. Sketch a graph of $y = \tan^{-1} x$. The line $y = x$ is shown dotted to help you. Be sure to indicate any asymptotes!



Homework Questions

- Evaluate the following. Be sure to express your answers in radians.
(a) $\sin^{-1}(0)$ (b) $\sin^{-1}(1)$ (c) $\sin^{-1}(-1)$ (d) $\cos^{-1}(0)$ (e) $\cos^{-1}(1)$
(f) $\cos^{-1}(-1)$ (g) $\tan^{-1}(0)$ (h) $\tan^{-1}(1)$ (i) $\tan^{-1}(-1)$
- In which quadrants do each of the following statements hold?
(a) $\sin \theta > 0$ and $\cos \theta < 0$ (b) $\tan \theta > 0$ (c) $\tan \theta < 0$
(d) $\sin \theta < 0$ and $\cos \theta > 0$ (e) $\cos \theta < 0$ and $\tan \theta > 0$
- For each of the following, solve the equation for the given range. Be sure to express your answers in radians.
(a) $\cos(\theta) = \frac{1}{\sqrt{2}}$ for $0 \leq \theta \leq 4\pi$ (b) $\sin(\theta) = \frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
(c) $\tan(\theta) = -\frac{1}{\sqrt{3}}$ for $-2\pi \leq \theta \leq 0$ (d) $\cos(\theta) = 0.42$ for $0 \leq \theta \leq 2\pi$
(e) $\sin(\theta) = 0.91$ for $-2\pi \leq \theta \leq 2\pi$ (f) $\tan(\theta) = 2.14$ for $0 \leq \theta \leq 5\pi$
- For each of the following, solve for all x with $0 \leq x \leq 2\pi$. Express all your answers in radians and give exact answers if possible.
(a) $2 \cos x = 1$ (b) $\tan x = \sqrt{3} - 2 \tan x$ (c) $3 \sin^2 x + 4 = 5$
(d) $4 \tan x + 3 = 2$ (e) $3 \cos^2 x + 2 = 3 - 2 \cos x$ (f) $3 \sin^2 x + 3 \sin x + 4 = 3 - 2 \sin x$