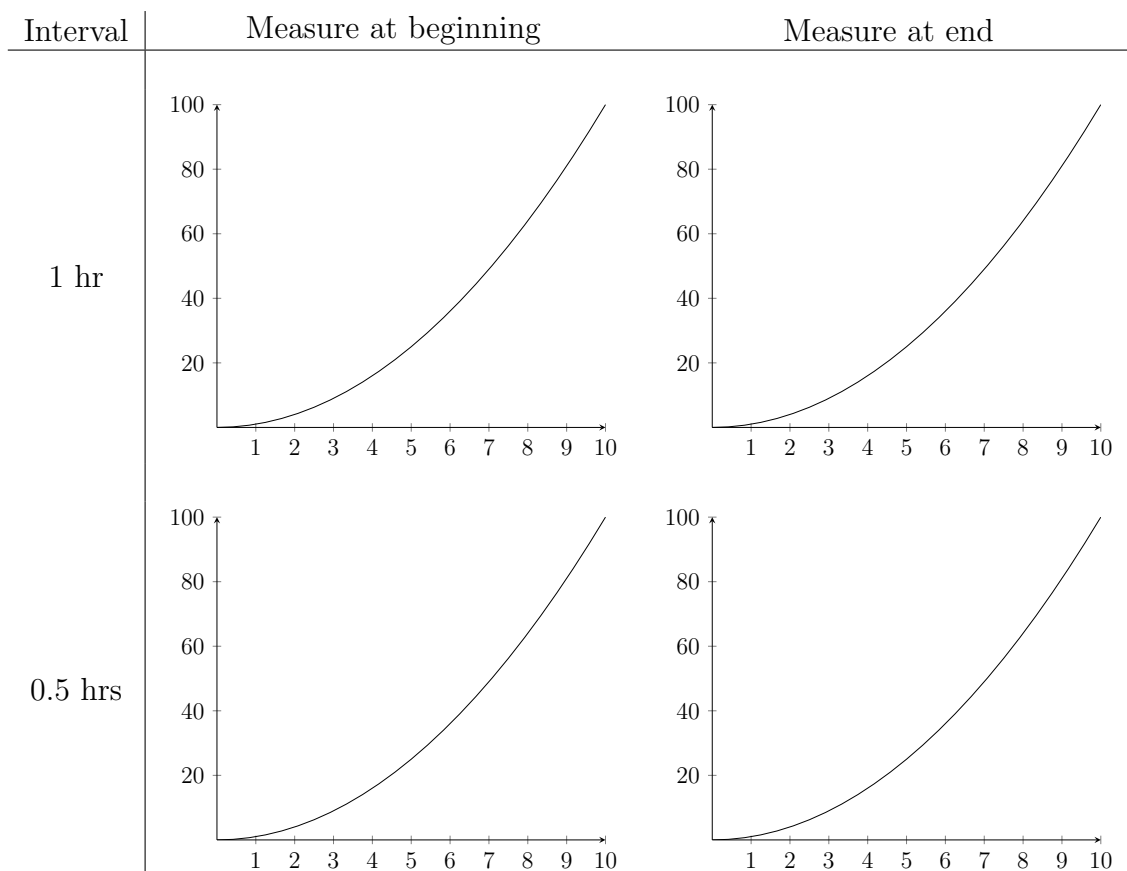


From Last Time

- We hypothesized that the area under a velocity curve between two time points gives the distance traveled between those two time points.
- We looked at a velocity curve and estimated the distance traveled between $t = 2$ hours and $t = 7$ hours by using measurements taken at fixed time subintervals:
 - Initially, we used one hour subintervals, then 0.5 hours.
 - We ‘measured’ velocity in two different ways: once at the beginning of each subinterval, and once at the end of each subinterval.

1. On each of the following copies of the curve $v(t) = t^2$, draw the relevant rectangles:



- (a) Which of your estimates are greater than the true distance traveled? Less?
- (b) Are your estimates in the first row more accurate than the second, or the second more accurate than the first? Explain in terms of what you’re actually trying to measure!

Digging a Little Deeper

2. Is it always the case that measuring at the beginning of each subinterval gives you an underestimate of the area, and measuring at the end gives you an overestimate? Can you draw a curve where it would be the other way around? Draw two copies of such a curve and show the two different sets of rectangles on them.
3. Fill in the following:
- If a curve is _____, then measuring at the beginning of each subinterval _____ estimates the area under the curve, and measuring at the end of each subinterval _____ estimates the area.
 - If, on the other hand, a curve is _____, then measuring at the beginning of each subinterval _____ estimates the area under the curve, and measuring at the end of each subinterval _____ estimates the area.
 - In other words:
 - if $f'(x) \underline{\hspace{1cm}} 0$ then measuring at the beginning of each subinterval overestimates the area under the curve, and measuring at the end of each subinterval underestimates the area.
 - If $f'(x) \underline{\hspace{1cm}} 0$ then measuring at the beginning of each subinterval underestimates the area under the curve, and measuring at the end of each subinterval overestimates the area.

A Bit of Notation

Over the next couple of days, we'll be developing notation to allow us to accurately define and compute areas under curves.

Width of Subintervals We will call the width of each subinterval Δx (or Δt if we're dealing with time). Read this as 'delta x' (or 'delta t'). Suppose you want to find the area under the curve between $x = a$ and $x = b$.

4. (a) What is the width of the interval from $x = a$ to $x = b$?
- (b) If we're dividing that interval into n equal subintervals, then $\Delta x = \underline{\hspace{2cm}}$.
- (c) To get a better estimate of the area, what do we do to n ? What happens to Δx ?

A Bit of Terminology

Notice that the beginning of each subinterval is its left-hand side, and the end is its right-hand side.

We will therefore call an area estimated by taking measurements at the beginning of each subinterval a *Left-Hand Sum* (or LHS). We call an area estimated by taking measurements at the end of each subinterval a *Right-Hand Sum* (or RHS).

The Heights of Rectangles

5. Suppose for now that we are using the LHS to estimate the area under the curve $f(x)$ over the interval from $x = a$ to $x = b$, with n equal subintervals.

Describe in your own words how to find the *heights* of each of your rectangles. Your description should include: where to start, how to find the height of the first rectangle, where to move to, how to find the height of the second rectangle, how to continue, and when to stop.

You should use the symbol Δx in your description. Feel free to draw a picture or two if they help.

A Bit More Notation: A Sequence of Subintervals

Suppose we let a be the beginning of the interval over which we aim to measure the area under a curve $f(x)$. Let the end of the interval be b , and fix a number n of subintervals. Then $\Delta x = \underline{\hspace{2cm}}$ is the width of each subinterval.

6. (a) Left-hand of 1st subinterval is .
- (b) Starting from a , how far do you have to move along the x -axis to get to the left-hand of the second subinterval?
- (c) In terms of a and Δx , write down a formula for the left-hand of the second subinterval:

Left-hand of 2nd subinterval is .

(d) Starting from a , how far do you have to move along the x -axis to get to the left-hand of the third subinterval?

(e) In terms of a and Δx , write a formula for the left-hand of the third subinterval:

Left-hand of 3rd subinterval is _____.

(f) Starting from a , how far do you have to move along the x -axis to get to the left-hand of the i^{th} subinterval?

(g) Continuing the pattern, in terms of a and Δx , write a formula for the left-hand of the i^{th} subinterval:

Left-hand of i^{th} subinterval is _____.

(h) i. How many subintervals are there?

ii. Write a formula for the left-hand of the last subinterval in terms of a and Δx .

iii. Write down a formula for the right-hand of the last subinterval.

iv. What should the right-hand of the last subinterval be? Check that your formula from 6(h)iii gives this!

Putting It All Together: Back to Areas of Rectangles

7. Suppose we divide the interval $[a, b]$ into n equal subintervals, each of length $\Delta x =$ _____. Then the left-hand of the i^{th} subinterval is at $x =$ _____, and the right-hand of the i^{th} subinterval is at $x =$ _____.

8. For a LHS:

- The height of the first rectangle is _____. Its width is _____. Therefore its area is _____.
- The height of the second rectangle is _____. Its width is _____. Therefore its area is _____.
- If $n = 5$, write down the total estimated area in terms of symbols developed above:

9. For a RHS:

- The height of the first rectangle is _____. Its width is _____. Therefore its area is _____.
- The height of the second rectangle is _____. Its width is _____. Therefore its area is _____.
- If $n = 5$, write down the total estimated area in terms of symbols developed above:

10. By filling in the following blanks and table, estimate the area under the curve $f(x) = \frac{1}{x}$ between $x = 1$ and $x = 4$ using $n = 5$ subintervals using a LHS, then again using a RHS. Use all the notation above throughout your calculations and draw pictures illustrating your work.

$$a = \text{_____} \quad b = \text{_____} \quad \Delta x = \text{_____}$$

$$1^{\text{st}} \text{ interval: } \text{_____}; \quad 2^{\text{nd}} \text{ interval: } \text{_____}; \quad 3^{\text{rd}} \text{ interval: } \text{_____};$$

$$4^{\text{th}} \text{ interval: } \text{_____}; \quad 5^{\text{th}} \text{ interval: } \text{_____}.$$

	$a =$ _____	_____	_____	_____	_____	_____	$= b$
Width	----- ----- ----- ----- ----- -----						
LHS Ht	----- ----- ----- ----- ----- -----						
RHS Ht	----- ----- ----- ----- ----- -----						
LHS Area	----- ----- ----- ----- ----- -----						
RHS Area	----- ----- ----- ----- ----- -----						

$$\text{LHS} \approx \text{_____} \quad \text{RHS} \approx \text{_____}$$

What's Next

For larger n , writing down the sums and calculating them gets very tedious. This week in lab, we've been developing notation to make writing down such sums easier. Next week in lab, we will use calculators to estimate sums for much larger n . Between the two labs, we'll develop yet more notation, and eventually find a way to compute the areas under certain curves precisely.