Review

1. (a) The Chain Rule states that:

$$\frac{d}{dx}\left[f(g(x))\right] =$$

(b) By taking the antiderivatives of both sides, we get:

$$\int f'(g(x))g'(x)\,dx =$$

Method of u-substitution

2. (a) Suppose we want to evaluate $\int f'(g(x))g'(x)\,dx$. Let $u = g(x)$.

Then $du =$ _____ dx.

(b) Hence,

$$\int f'(g(x))g'(x)\,dx =$$
$$=$$
$$=$$

(c) Note also that

$$\int_a^b f'(g(x))g'(x)\,dx =$$
$$=$$

Examples

Note: Not all of these require substitutions. One of the most important integration skills is spotting what is the easiest method to use for a given integral.

3. $\int x(1+x^2)^5\,dx$

4. $\int_0^{\frac{\pi}{2}} \sin x \cos x \; dx$

5. $\int_0^1 \frac{x}{\sqrt{x^2+1}} \; dx$

6. $\int \frac{1}{3x+1} \; dx$

7. $\int \frac{1}{e^{3x}} \; dx$

8. $\int \sin^2 x \, dx$ (Hint: $\cos 2x = 1 - 2\sin^2 x$)

9. $\int_0^2 \frac{e^x}{1+e^{2x}} \, dx$ (Hint: What's another way to write e^{2x}? Rules of exponentials...)

10. $\int \frac{\ln x}{x} \, dx$

11. $\int \cos^2 x \sin x \, dx$

12. $\int xe^{-x^2}\,dx$

13. $\int \tan x\,dx$ (Hint: write $\tan x$ as a quotient of two other trig functions.)

14. $\int \frac{2x}{\sqrt{1-x^4}}\,dx$

15. $\int_0^1 x\sqrt{x+1}\,dx$ (Hint: Let $u = x + 1$. Then $x =$...?)