

Newton's Law of Cooling

Newton formulated the principle that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surroundings.

- (a) Suppose that the temperature of an object at time t is given by $T(t)$. Denote the surrounding temperature T^* , and assume it is constant. Write down a differential equation expressing Newton's Law of Cooling:

$$\frac{dT}{dt} = \underline{\hspace{10em}}$$

Note that if the surrounding temperature is greater than the temperature of the object, we expect the temperature of the object to , whereas if the surrounding temperature is lower than the temperature of the object, we expect the temperature of the object to . Given that we usually take our constants of proportionality to be positive, does your equation above reflect this? If not, correct it.

- (b) What is the equilibrium solution to your differential equation? By sketching a slopefield (pick values of k and T^* if needed), determine if the equilibrium is stable or unstable.

- (c) Plot two solutions of the equation on your slopefield. One with $T(0) > T^*$, and one with $T(0) < T^*$. What is the behavior of the solutions as $t \rightarrow \infty$? Why does this make sense?

Mixing Problems

Container problems are another class of situations we can model using the kind of differential equations we've been studying. The idea is this: if we have a container into which we add a substance at a certain rate and remove the substance at another rate, then the rate of change of the amount of the substance in the container is given by the difference between the two rates. That is:

$$\text{rate of change} = \text{rate in} - \text{rate out}$$

5. An 800 gallon tank is filled with brine which contains 300 pounds of salt. Every minute two gallons of dilute brine containing 0.1 pound of salt per gallon are pumped in and two gallons of the (well-mixed) brine are pumped out. Find a differential equation and initial condition satisfied by $S(t)$, the weight (in lbs) of salt in the tank at time t . Solve the initial value problem to find $S(t)$. Draw a graph of $S(t)$ and describe its behavior as $t \rightarrow \infty$. Explain why your answer makes sense.

6. A tank of salt water has 100 pounds of salt dissolved in 1000 gallons of water. Every minute four gallons of salt water having 0.5 pounds of salt per gallon are pumped in and four gallons of the (well-mixed) salt water are pumped out. Find a differential equation and initial condition satisfied by $S(t)$, the weight (in lbs) of salt in the tank at time t . Solve the initial value problem to find $S(t)$. Draw a graph of $S(t)$ and describe its behavior as $t \rightarrow \infty$.