Newton's Law of Cooling

Recall that Newton's Law of cooling says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature (denoted $T(t)$) and the temperature of the surroundings (T^*).

1. Write this down as a differential equation.

In practice, we do not usually know the value of the constant k at the start of the problem. Instead, problems involving Newton's Law of Cooling are often expressed as *boundary value problems*. That is, we measure the temperature twice and use the second measurement to determine k.

2. The temperature in a certain Duke dorm is a stuffy 76°F at midnight when the power goes out. Outside the temperature holds steady at 35°F. The students measure the temperature in the dorm at 1 am and find that it has decreased to 65°F. Worried, they compute how low the temperature will fall by 7 am when Duke Energy Corporation has pledged that the heat will come back on. Recreate their calculation.

Often, loosening some our assumptions leads to much harder differential equations than the ones we saw on the previous worksheet. Below, we will see an example of this in each of the above two situations.

3. Suppose we have a coffee cup whose temperature is 110°F, like on the last worksheet. This time, though, instead of the room being at a fixed temperature, it is air conditioned. Its temperature varies sinusoidally between 67°F and 73°F with a period of 2 hours. Suppose that at time $t = 0$ the temperature of the room is 70°F (note: that is the midline!).

   (a) T^* is a function of time. Write such a function down. (Refer to the Modeling with Trig Functions lab if you need a reminder.)

   (b) Assuming that $k = 0.7$ as on the last worksheet, write down an initial value problem for this situation.

   (c) Your equation is not separable. Use Euler's method in a spreadsheet to model the solution. Take $\Delta t = 0.1$ hours and compute data points for 15 hours. Plot your solution and sketch a graph below.

   (d) After a while, the temperature settles into what appears to be a sinusoidal pattern with period 2 hours. Set your y-axis to be between 67°F and 73°F. You should see that the amplitude of the oscillation is not 3°F, like the amplitude of the temperature in the room. Explain why that's the case in terms of how you'd expect the temperature of the coffee to respond to changes in the ambient temperature.

Mixing Problems

Recall: we have a container into which we add a substance at a certain rate and remove the substance at another rate, then the rate of change of the amount of the substance in the container is given by the difference between the two rates. That is:

$$\text{rate of change} = \text{rate in} - \text{rate out}$$

4. Suppose that we are in the situation as in the last question of the previous worksheet: A tank of salt water has 100 pounds of salt dissolved in 1000 gallons of water, except that 2 pounds of salt *but no water* is added per minute. As before, 4 gallons of brine are pumped out each minute.

    (a) What will be the volume of liquid in the tank t minutes after the start? What will be the concentration of salt in the tank (in terms of S and t)?

    (b) Write down (but do not try to solve) an initial value problem for $S(t)$.

    (c) How long can this situation continue for? (Hint: look at the denominator of your equation above!)

    (d) Use Euler's method in a spreadsheet to describe the behavior of $S(t)$. Pick a very small Δx to get good results. Model your solution until the maximum time you found above. Sketch a graph of $S(t)$ below.

    (e) In practice, is this a good model for the entire time period? Why not? (Hint: think about solubility!)